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Design of Maximally-Flat GCF Compensation Filter

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Abstract-The design of the compensation filter of a generalized comb filter (GCF) using Maximally-Flat minimization is presented. The coefficients of the proposed compensation filter are obtained by solving two linear equations. The filter operates at a low rate and considerably reduces the passband droop of the GCF filter.

Index Terms-GCF filter, Maximally-Flat, FIR filters.

1. INTRODUCTION

The simplest decimation filter is the cascaded-integrator comb (CIC) filter, which is described in [1]. Unfortunately, this filter has a high passband droop and low stopband attenuation. In order to improve the passband as well as the stopband characteristics of a CIC filter, different methods are proposed, for example, [2]-[6]. A generalized CIC decimation filter (GCF) is introduced in [6] to improve the stopband attenuation as well as the spanned bands around the zeros of the CIC filter (folding bands).that is, the frequency points $2\pi k/D$ where D is the decimation factor, and k=1,….., D-1.
The transfer function of the GCF filter is expressed as,

$$H_{GCF_N}(z) = \prod_{n=1}^{N} \frac{\sin(\alpha_n/2)}{\sin(\alpha_n D/2)} \prod_{n=1}^{N} \frac{1-z^{-D}e^{-j\alpha_n D}}{1-z^{-1}e^{-j\alpha_n}} \quad (1)$$

Where D stands for the decimation factor and α_n, n = 1,…,N, are rotational parameters optimized such that the minimum attenuation within folding bands is maximized.
The discrete- time Fourier transform (DTFT) of $H_{GCF_N}(z)$is

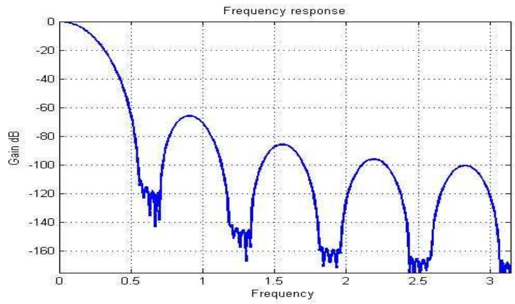
$$H_{GCF_N}(e^{j\omega}) = H(\omega) \exp\left(-j\frac{(D-1)}{2}\left(\omega N + \sum_{n=1}^{N} \alpha_n\right)\right) \quad (2)$$

where

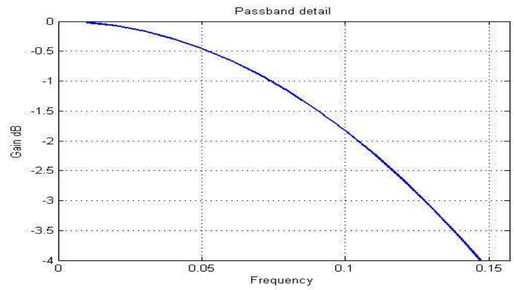
$$H(\omega) = \prod_{n=1}^{N} \frac{\sin(\alpha_n/2)}{\sin(\alpha_n D/2)} \prod_{n=1}^{N} \frac{\sin((\omega+\alpha_n)D/2)}{\sin((\omega+\alpha_n)/2)} \quad (3)$$

Note that, generally the filter $H_{GCF_N}(z)$ has linear-phase characteristics and complex- valued coefficients .The real-valued filter coefficients of $H_{GCF_N}(z)$ are obtained satisfying$\alpha_n = \alpha_{N-n}$.A useful choice for α_n is $\alpha_n = q_n\pi/vD$, where v is a positive integer and$q_n$is a real value in the range [-1,1] [6]. With α_n=0, n=1,……,N , traditional CIC filter is obtained.

Now, we consider a GCF design example using the following parameters N=6,D=10,$v = 4$ and q_n= [-0.54,-0.92 , 0, 0.92, 0.54] [6]. Figure 1(a) shows the magnitude response of the resulting filter, while the passband detail is illustrated in Fig. 1(b). Notice the increased width and attenuations at the folding bands. Unfortunately, the GCF filter exhibits a high passband droop (see Fig. 1(b)).



(a)



(b)

Fig.1. Magnitude response of the GCF for $N = 5, D = 10$ *and* $v = 4$.

2. GCF COMPENSATION FILTER

The transfer function of the proposed GCF compensation filter is

$$P(z^D) = a + bz^{-D} + az^{-2D} \quad (4)$$

Where a and b are real-valued constants .The compensation filter is cascaded with the GCF, as shown in Fig. 2(a). Using the multirate identity [10], filter P(z^D) can

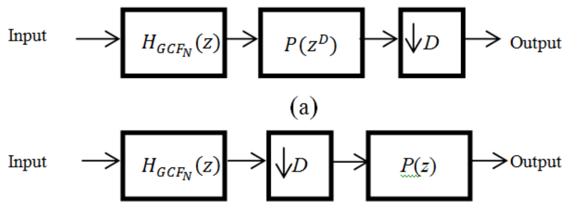


Fig.2. Decimation block diagram.(a) Generalized CIC Filter and compensation filter.(b) Efficient structure for decimation.

be moved to a lower rate, resulting in a more efficient structure shown in Fig. 2(b).The cascade of the compensation filter P(z^D) and the GCFyields the following overall transfer function:

$$G(z) = H_{GCF}(z)P(z^D) \tag{5}$$

By performing the DTFT, equation (5) becomes

$$G(e^{j\omega}) = e^{-\frac{j\omega((D-1)N+2D)}{2}} H(\omega)P_R(D\omega) \tag{6}$$

Where $P_R(D\omega)$ is the amplitude response of $P(e^{j\omega D})$, which is given by

$$P_R(D\omega) = b + 2a\cos(D\omega) \tag{7}$$

Next is find out the coefficients a and b.We define the error function in the frequency range $[0,\omega_p]$, where ω_p is the upper edge frequency of the signal band, which is necessary to preserve after decimation, i.e.

$$E(\omega) = H(\omega)P_R(D\omega) - 1 \tag{8}$$

In order to find coefficients a and b, we impose the condition in which the error function should be zero at frequencies ω = 0and ω = ω_0, where ω_0 is less than or equal to ω_0.For ω = 0, from (3), (6)–(8), it follows that

$$2a + b = 1 \tag{9}$$

Similarly, for $\omega = \omega_0$ (see (6)-(8)), the imposed condition results in

$$H(\omega_0)(2a\cos(D\omega_0) + b)=1 \tag{10}$$

Solving equations (9) and (10), the values of a and b respectively are given as

$$a = \frac{1}{2}\frac{1-\frac{1}{H(\omega_0)}}{1-\cos(D\omega_0)} \tag{11}$$

$$b = 1 - 2a \tag{12}$$

Equations (11) and (12) are general equations, where the value of $\omega_0$depends on the method used for the Minimization of error,as described in the following. Here, we consider error function E(ω) to be maximally flat at ω = 0, i.e., the error function has as many derivatives that are vanishing at ω = 0 as possible .Since the error function is an even function of ω, its odd indexed derivatives evaluated at ω = 0 are automatically zero. Therefore, it follows that

$$\frac{d^2E(\omega)}{d\omega}, \omega\to 0 = 0 \tag{13}$$

After small computations from (9), (10) and (13), we

$$a = \left.\frac{H''(\omega)}{2D^2}\right|_{\omega=0} \tag{14}$$

get, where $H''(\omega)$is the second derivative of H(ω)with respect to ω. Now, we obtain a closed-form equation for first derivative of H(ω). First, consider the following derivative.

$$\frac{d}{d\omega}\left\{\frac{\sin((\omega+\alpha_n)D/2)}{\sin((\omega+\alpha_n)/2)}\right\} = \frac{\sin((\omega+\alpha_n)D/2)}{\sin((\omega+\alpha_n)/2)} \times\left(\frac{D}{2}\cot((\omega+\alpha_n)D/2) - \frac{1}{2}\cot((\omega+\alpha_n)/2)\right) \tag{15}$$

Using (3), (15), and the product rule for derivatives, the first derivative of H(ω),

$$H'(\omega) = \frac{H(\omega)}{2} \times \sum_{n=1}^{N}\left(D\cot\left(D\frac{\omega+\alpha_n}{2}\right) - \cot\left(\frac{\omega+\alpha_n}{2}\right)\right) \tag{16}$$

Notice that the evaluation of above equation at ω = 0 is equal to zero since $\alpha_n = \alpha_{N-n}$ and cot(·) is an odd function. Therefore,

$$H''(\omega) = \frac{H(\omega)}{4}\left[\sum_{n=1}^{N}\left(\csc^2\left(\frac{\omega+\alpha_n}{2}\right) - D^2\csc^2\left(D\frac{\omega+\alpha_n}{2}\right)\right) + \left(\sum_{n=1}^{N}\left(D\cot\left(D\frac{\omega+\alpha_n}{2}\right) - \cot\left(\frac{\omega+\alpha_n}{2}\right)\right)\right)^2\right]. \tag{17}$$

Substituting ω = 0 into (17) and (14),we get the value of a as given in Eq.(19).Thus, solving (13) and using (12), the value of a is given by:

$$a = \frac{1}{8D^2}\sum_{n=1}^{N}\left(\frac{1}{\sin^2(\alpha_n/2)} - \frac{D^2}{\sin^2(\alpha_n D/2)}\right). \tag{18}$$

For the traditional CIC compensation filter (α_n=0 for n = 1, . . . , N), the value of a reduces to

$$a = \frac{N(1-D^2)}{24D^2} \tag{19}$$

It can easily be shown that the coefficient a for the maximally flat case defined in (18) can also be obtained from general equation (11) by replacing ω_0 with zero and applying the L'Hôspital rule.

3. DICUSSION AND RESULTS

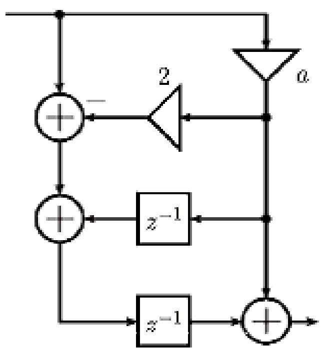


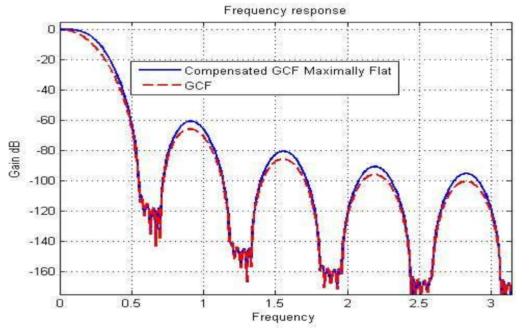
Fig. 6. Compensation filter structure *P(z)*. (a) Maximally flat

Observe that the structure shown in Fig. 6 requires one multiplier and three adders. In the following we analyze the passband droop R_P after compensation.
The compensation filter in [4] is a multiplierless filter with only three adders. This filter exhibits better compensation in narrowband and has less computational load than [3], whereas the method in [3] has better results in wideband compensation(for more details, see [4]).We illustrate the proposed design with one example.

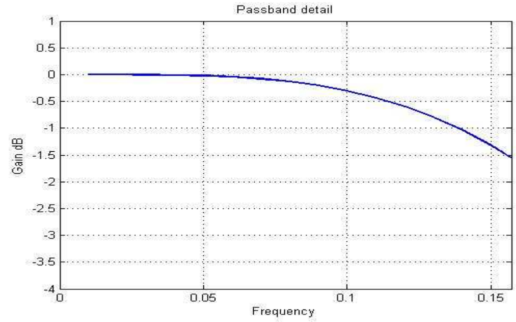
Example. Consider the design of a GCF compensation filter with the following parameters*:* D = 10, N = 5, ω_p =0.45 π/D, and α_n = $q_n\pi$/4D for n = 1, 2, 3, 4, 5, where q_n=[−0.54,−0.92, 0, 0.92, 0.54] [6]. The corresponding passband droop and stopband attenuation of the GCF are Ap = −3.71 dB and As = −66.15 dB, respectively, as shown in Fig.7.For maximally flat case. Using (18), it follows that a = −0.20765 and b = 1.42911. The passband droop after compensation is −1.08dB. Similarly, the stopband attenuation becomes −61.01dB.

TABLE 1
PARAMETERS IN THE DESIGN EXAMPLE

PARAMETERS	GCF_6	MAXIMALLY FLAT
PassbandAttenuation (A_P)	**-3.71**	**-1.08**
StopbandAttenuation (A_S)	**-66.15**	**-61.01**
a		**-0.20765**
b		**-1.42911**
ω_0		**0**



(a)



(b)

Fig.7. Overall magnitude response of the GCF and the compensation filters in the design example.

4. CONCLUSIONS

A novel approach for a GCF compensation filter design has been presented. The technique is based on a 2D-ordercompensation filter, which becomes a second-order filter after moving to a low rate. The proposed method includes the maximally flat designs. Considering that the maximally flat design need only one multiplier and could be a good choice for narrow passband compensation. However, for wide-passband compensation, the best choice is the minimax design, which requires two multipliers.

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